A Short Illustration of "When to Make an Introduction?"

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1 Motivation

The literature in network formation typically assumes that the creation and severing of a relationship is a decision made only by the two agents involved. This paper broadens the scope of investigation by analyzing a model where links can be formed via introductions from mutual friends.

2 The Model

- There is a set of players endowed with different levels of ability.
- They are situated in a network but can modify its structure by making introductions (create links for pairs of neighbours).
- They then undergo a matching process upon the network: they pair with one of their neighbours or stay alone. They always prefer a more capable partner and the worst outcome is to remain single. There is a unique stable matching of this process for any network structure and strict ability ranking.
- Players make introductions to maximize their matching outcome.
- I investigate how the incentive of introduction depends on the ability configuration and network positions.

3 An Example

Figure 1 gives an example of how a player would act under a specific circumstance. Figure 1(a) pictures a network for 7 players: let player x be endowed with ability x. Figure 1(b) then indicates the unique stable matching under the status quo: the most capable player 7 matches with his strongest neighbour 2, the following best player 6 pairs with his optimal available choice 5, 4 stays with 1, and 3 remains alone. Figure 1(c) shows three possible introductions player 1 can make: for 3 and 4, for 7 and 3, and for 7 and 6. Figure 1(d), (e), (f) then show the matching outcome after the three introductions respectively.

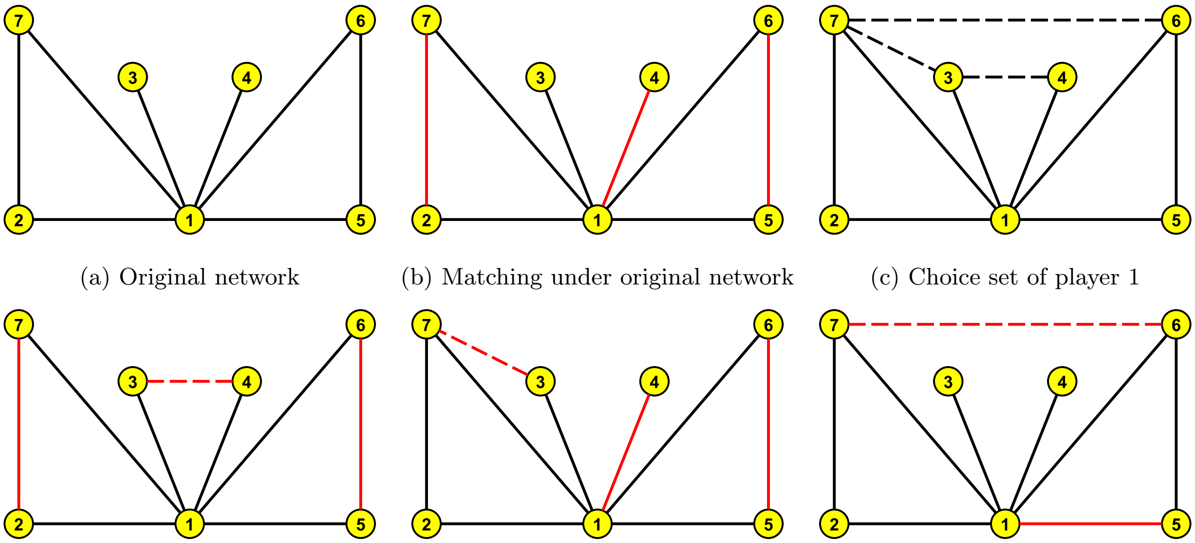
We can see that in the first case, player 1 is worse off as he loses his original match and needs to stay single. In the second case, player 1's matching is unaffected. In the third case, player 1 gets better off with a new match to player 5. So, player 1 will introduce 7 and 6 and is indifferent on whether to link 7 and 3.

Our goal is to provide a systematic account of when the three outcomes: worse off, unaffected, and better off will happen for a general endowment configuration and network structure.

4 Results

- When one or both of the neighbours being introduced is(are) less capable than the introducer, the introducer's match will be unaffected for whatever network position he is at.
- Let an alternating path be a path where links along it facilitate match and unmatch alternatively, then the condition for a player to be affected by making an introduction is that (1) both players he introduces are stronger than him and (2) there exists an alternating path from a player he introduces to him in the original network. [1]

[1] To define alternating path with the language of math: a network is represented by $g = \{g_{ij}\}_{ij \in N}$ where $g_{ij} \in \{0, 1\}$. An alternating path from i to j in g is a sequence of distinct nodes $i = v_0, v_1, ..., v_m = j$ such that $g_{v_{k-1}v_k} = 1$ for all $k = 1, ..., m$ and all pairs of players v_{k-1} and v_k are matched while all pairs v_k and v_{k+1} are unmatched under g for $k = 1, 3, ...$.



(a) Original network (b) Matching under original network (c) Choice set of player 1

(d) Outcome 1: worse off (e) Outcome 2: unaffected (f) Outcome 3: better off

Figure 1

- The length of the alternating path specified above determines the direction of the effect: an odd one for impairment and an even one for improvement. Again we can refer to Figure 1 for an illustration. Player 1 can make an introduction for player 3 and 4, who are both stronger than him. Figure 1(b) shows that there is an odd-length alternating path from player 4 to 1 in the original network: {4,1}, and the introduction makes player 1 worse off. In the meanwhile, player 1 can also make an introduction for player 7 and 6, who are both stronger than him. Figure 1(b) indicates an even-length alternating path from player 6 to 1: {6,5,1}, and the introduction makes player 1 better off.
- I can characterize the stability of a network based on robustness to introductions.
- Introduction, though always amplifies the feasible matching set for players, is not necessarily welfare improving. This is true for multiple welfare measures. For example, in the case of Figure 1, an introduction between 7 and 6 leads to a decrease in the total number of matches from 3 to 2.

5 Discussion

This paper demonstrates a way of strategic thinking under a particular network setup. Agents can think fast: an introduction that involves a weaker player is always harmless, or slow: the decision on whether to introduce two strong players calls for a careful examination of the network structure.

I intend to extend the analysis in the following directions:

(1) I use a particular one-to-one matching procedure to define the network utility. It is a special case of when players share scarce resources in a network. It should not be hard to generalize the model to the following setting. Assume that players who are heterogeneous in their attractiveness seek to exchange favours and information with capable neighbours but have limited attention to allocate. A network and an endowment profile would again define a stable attention sharing profile that pins down the utility of players. We can analyze how players compete for the scarce network resources by manipulating the network structure.

The strategic reasoning we obtain with the simple model should still apply since there is a similar flow of changes given a new link. The effect will pass through a sequence of players who alternatively belong to the following two kinds: one obtaining better resources and let go the inferior, another being dumped and have to find new partners originally unqualified.

(2) One channel of having the clustering feature in a network is via "meeting friends of friends", which is likely to be accompanied by introductions. Investigation on clustering typically assumes that the meeting happens at random. A strategic analysis of introduction incentives may shed light on specific features of clustering that cannot be explained by the random process.